Diploma Programme
Programme du diplôme
Programa del Diploma

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Mathematics: analysis and approaches
Higher level
Paper 3

20 May 2026

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

1 hour 15 minutes

Instructions to students

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[55 marks]**.

2226 – 5027

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Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 30]

This question asks you to investigate some graphical and algebraic properties of a family of quartics of the form $f(x) = x^4 + bx^2 + d$.

The function f is defined by

$$f(x) = x^4 + bx^2 + d, \text{ where } b \in \mathbb{R} \text{ and } d \in \mathbb{R},\ d \neq 0.$$

(a) Consider the case where $b = 1$ and $d = 1$.

(i) Sketch the curve $y = x^4 + x^2 + 1$, labelling any axes intercepts. [2]

(ii) Hence, state why the equation $x^4 + x^2 + 1 = 0$ has no real roots. [1]

(b) Consider the case where $b = -2$ and $d = 1$.

Sketch the curve $y = x^4 - 2x^2 + 1$, labelling any axes intercepts. [2]

In parts (c) to (g), consider

$$f(x) = x^4 + bx^2 + d, \text{ where } b \in \mathbb{R} \text{ and } d \in \mathbb{R},\ d \neq 0.$$

(c) (i) Write down an expression for $f'(x)$. [2]

(ii) Show that f' is an odd function. [3]

A root α is defined as a **double root** of an equation $f(x) = 0$ if both $f(\alpha) = 0$ and $f'(\alpha) = 0$.

(d) State a feature of the graph of $y = f(x)$ that indicates whether the equation $f(x) = 0$ has a double root α. [1]

In parts (e) to (g), consider the case where the equation $f(x) = 0$ has a double root α.

(e) Show that $-\alpha$ is also a double root of the equation $f(x) = 0$. [4]

(This question continues on the following page)

(Question 1 continued)

(f) (i) By considering $f'(\alpha) = 0$, or otherwise, show that $\alpha^2 = -\frac{b}{2}$. [3]

(ii) Hence, show that $d = \frac{b^2}{4}$. [2]

(g) Deduce the set of values of b for which the equation $f(x) = 0$ has

(i) real roots; [1]

(ii) purely imaginary roots. [1]

In parts (h) and (i), consider

$$f(x) = x^4 + bx^2 + d\text{, where } b \in \mathbb{R},\ b < 0 \text{ and } d \in \mathbb{R},\ d \neq 0.$$

(h) Show that the solutions to the equation $f''(x) = 0$ are $x = \pm\sqrt{-\frac{b}{6}}$. [3]

The curve $y = f(x)$ has a point of inflexion at $x = \sqrt{-\frac{b}{6}}$.

The equation of the tangent to the curve $y = f(x)$ at $x = \sqrt{-\frac{b}{6}}$ has the form

$$y = -\frac{2\sqrt{6}}{9}(-b)^{\frac{3}{2}}x + c.$$

(i) Show that $c = \frac{b^2}{12} + d$. [5]

2. [Maximum mark: 25]

This question asks you to investigate the motion of a particle moving back and forth in a horizontal line between two end points.

A particle moves back and forth in a horizontal line between two end points.
A fixed origin O is the midpoint between these end points.
The particle's displacement, x metres, from O at time t seconds is given by

$$x = 4\sin\left(2t + \frac{\pi}{3}\right), \text{ for } 0 \le t \le \pi.$$

(a) Determine

(i) the amplitude of the particle's motion; [1]

(ii) the particle's initial displacement from O; [2]

(iii) the value of t when the particle first passes through O. [2]

Now consider the general case of a particle moving back and forth in a horizontal line between two end points.

The particle's acceleration is always directed towards a fixed origin O at the midpoint of these end points.

The particle's acceleration, a, at a displacement, s, from O satisfies the differential equation

$$a = -n^2 s, \text{ where } n > 0.$$

The particle's displacement, s, from O at time t is given by

$$s = A\sin(nt + b), \text{ where } t \ge 0,\ A,\ n > 0 \text{ and } -\pi \le b \le \pi.$$

(b) By finding expressions for $\frac{\mathrm{d}s}{\mathrm{d}t}$ and $\frac{\mathrm{d}^2 s}{\mathrm{d}t^2}$, verify that $s = A\sin(nt + b)$ satisfies the differential equation $a = -n^2 s$. [2]

(This question continues on the following page)

(Question 2 continued)

(c) (i) Use the chain rule to show that $a = v\dfrac{\mathrm{d}v}{\mathrm{d}s}$, where v is velocity. [1]

(ii) By solving the differential equation, $v\dfrac{\mathrm{d}v}{\mathrm{d}s} = -n^2 s$, show that $v^2 = n^2(A^2 - s^2)$. [5]

(iii) Hence, or otherwise, find the particle's maximum speed. [2]

The continuous random variable S denotes the particle's displacement, s, from O at time t.

The probability density function f of S is defined by

$$f(s) = \begin{cases} \dfrac{1}{\pi\sqrt{A^2 - s^2}}, & -A < s < A \\ 0, & \text{otherwise.} \end{cases}$$

(d) Show that $\mathrm{P}\left(-\dfrac{A}{2} \le S \le \dfrac{A}{2}\right) = \dfrac{1}{3}$. [4]

For $-A < s < A$, the function $f(s)$ can be expressed in the form $\dfrac{k}{|v(s)|}$, where $k > 0$ and $v(s)$ is the particle's velocity at a displacement, s, from O.

(e) Find the value of k. [3]

(f) (i) Determine $\mathrm{E}(S)$, justifying your answer. [2]

(ii) Interpret the result found in part (f)(i) in the context of the particle's motion. [1]